SMA 335 Ordinary Differential Equations I Notes Sample Document 1dc065f1fbf404a0e3bd01bd2a6b1f71

Bsc. Civil Engineering (Kenyatta University)

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SMA 335
ORDINARY DIFFERENTIAL EQUATIONS I

BY

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DEPARTMENT OF MATHEMATICS

PREFACE

This module is designed primarily to provide the readers with the best preparation possible for the ordinary Differential Equations (SMA 335 ordinary Differential Equations I)

In its present form this module has developed from course given by the author over the last **thirty two** years in various universities to the audience of Mathematicians, Physicists and Engineers in the university of **Madras, Kenyatta University, University of Nairobi and Jomo Kenyatta University of Agriculture and Technology.**

This module, Ordinary Differential Equations I is compiled from the Author's Advanced Differential Equations, Oxford Publications, London and Nairobi. Most of the theory and problems are freely taken from the Author's Book for which the author has sole Copyright.

It is hoped that it will be of great interest to students of pure and Applied Mathematics following Ordinary Differential Equations I.

Each Chapter begins with a brief statement of definitions principles and important Theorems followed by a set of solved problems. Attention has been given to the Chapters on Applications of the theory.

The author is pleased to acknowledge Dr L. O. Odongo who worked through the entire manuscript and checked all the problems in each and every Chapter.

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LESSON 1
Differential Equations and Their Solutions

1.1 Introduction
The subject of differential equations constitutes a very important and useful branch of modern mathematics. In this lesson we shall consider some definition of ordinary differential equations.

1.2 Objectives of the lesson
By the end of this lesson you should be able to.
- define a differential equation
- define the order and the degree of a differential equation.
- form the differential equation and solve simple differential equations.

1.3 Definition of ordinary differential equation
An equation containing any derivatives such as $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, is called an ordinary differential equation.

Examples of differential equations

1. $\frac{dy}{dx} = 5x$
2. $\frac{dy}{dx} + 3x + 5 = 0$
3. $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 9y = 0$
4. $\frac{d^2y}{dx^2} + y = \sin x$

are ordinary differential equations.

1.4 Definition of Partial Differential Equations
A differential equation involving partial derivatives of a dependent variable, with respect to more than one independent variable, is called a partial differential equation.
For example,

$$\frac{\partial}{\partial x}(2x^2 + 5y^2 + 3xy^2) = 0$$

$$\frac{\partial}{\partial x}(\sin x + 5y\cos x + x^2y^3) = 0$$

$\frac{\partial^2}{\partial y^2}(x^3 + 3x^2y + y^2) = 0$ are called partial differential equations. This means when you differentiate a term with respect to x, y is treated as constant similarly x is treated as a constant when we differentiate a term with respect to y.

Then $\frac{\partial}{\partial x}(3y^2x^3) = 3y^2\frac{\partial}{\partial x}x^3 = 9x^2y^2$

$\frac{\partial}{\partial y}(7y^2 \sin x) = 7\sin x\frac{\partial}{\partial x}y^2 = 14y\sin x$

1.5 Meaning of $\frac{d^2y}{dx^2}, \frac{d^3y}{dx^3} \ldots\ldots\ldots \frac{d^ny}{dx^n}$

$\frac{d^2y}{dx^2}$ means that we differentiate $\frac{dy}{dx}$ again with respect to x.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right]$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left[\frac{d^2y}{dx^2}\right]$$

similarly $\frac{d^ny}{dx^n}$ is defined.

1.6 The order of a differential equation

The order of a differential equation is the order of the highest differential coefficient present in the equation.
For example the equation,

$$7\frac{d^3y}{dx^3} + 8\frac{d^2y}{dx^2} + 9\frac{dy}{dx} + 8y = e^x$$

contains, $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}$ the highest differential coefficient is $\frac{d^3y}{dx^3}$. Hence the order of the equation is three.

Consider $3\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{10} + 8y = 0$

The highest differential coefficient present in this equation is $\frac{d^2y}{dx^2}$. Hence the order of the equation is two.

1.7 The degree of a differential equation

The degree of a differential equation is the degree (or power) of the highest differential coefficient present, when the differential coefficient are free from radicals and fractional exponents.

For example consider the equation,

$$\frac{d^3y}{dx^3} + \left(\frac{d^2y}{dx^2}\right)^{10} + 3\left(\frac{dy}{dx}\right)^7 + 8y = 0$$

The highest order present in this equation is $\frac{d^3y}{dx^3}$. The degree of this $\frac{d^3y}{dx^3}$ is one. Hence the degree of the differential equation is just one.

Example 1
Consider,

$$\left(\frac{d^3y}{dx^3}\right)^5 + \left(\frac{d^2y}{dx^2}\right)^{11} + \left(\frac{dy}{dx}\right)^{20} + y = x$$

Here the highest differential coefficient is $\frac{d^3y}{dx^3}$ or the order of the equation is 3. The degree of this highest differential coefficient is five and hence the degree of this equation is five.

Example 2

Consider, $\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} + 5\frac{dy}{dx} + 3y = 0$

The highest differential coefficient present here is $\frac{d^2y}{dx^2}$. First it should be free from radicals and fractional exponents.

$$\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} = -5\frac{dy}{dx} - 3y$$

$$\left[\frac{d^2y}{dx^2}\right]^3 = \left[-5\frac{dy}{dx} - 3y\right]^2$$

Then the degree of the equation is three.
Consider,

$$\sqrt{3 + \left(\frac{dy}{dx}\right)^2} = 2x$$

Removing the radical it is written as $3 + \left(\frac{dy}{dx}\right)^2 = 4x^2$.

Hence the degree of the equation is two.

1.8 The solution of a differential equation

The solution of a differential equation is an equation between x and y, which will satisfy the differential equation.

Example 3

a) Show that $y = x^2$ is a solution of the equation, $3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 10x + 6$

b) Check whether $y = x^2 + c$ where c is any constant is also a solution of the equation.

Solution

Let $y = x^2$ (1)

$\frac{dy}{dx} = 2x$ (2)

$\frac{d^2y}{dx^2} = 2$ (3)

substituting (1), (2) and (3) in the given equation
$3(2) + 5(2x) = 6 + 10x$ thus the equation is satisfied

1.9 General and particular solution of a differential equation.

Consider $\frac{dy}{dx} = 2$ (1)

Then $dy = 2dx$

Integrating, $\int dy = \int 2dx$

$y = 2x + c$ where c is any arbitrary constant

we say that $y = 2x + c$ is the general solution of the equation (1)

Geometrically, $y = 2x + c$ is a family of straight lines parallel to $y = 2x$
The solution $y = 2x + c$, represents a family of straight lines. This family $y = 2x + c$ is called **the general solution** of the equation $\frac{dy}{dx} = 2$.

When c takes a particular value (say $c = 3$) we get a particular member of the family, $y = 2x + 3$. It is a particular solution.

A particular solution is one where a value is given to c.

In the above example, the order of the equation $\frac{dy}{dx} = 2$, is one. Hence the general solution of this equation contains one arbitrary constant c.

In general, the solution of an n^{th} order differential equation will have n arbitrary constants. For example the general solution of a second order equation will have **two** arbitrary constants. When we give particular values for these constants we get particular solutions of the equation.

Example 4

Consider the equation $\frac{dy}{dx} = 3x^2$

$dy = 3x^2\, dx$

integrating both sides,

$\int dy = 3\int x^2 dx$

$y = x^3 + c$

The general solution of the equation is a family of curves. When c takes the value, say $c = 2$, $y = x^3 + 2$ is a particular curve of the family and it is called a particular solution of the equation

1.10 Forming Differential Equations

Differential Equations are often formed by expressing certain scientific laws or relationships in mathematical terms. We can also form them by eliminating the constants in an equation. If n constants are eliminated we will get an n^{th} order equations.

Example 5

The scientific law states that the acceleration of a particle during upward motion under gravity is -g. Write down this law as a differential equation.

Solution

Velocity of a particle is $\frac{dx}{dt}$

Acceleration of the particle is $\frac{d^2x}{dt^2}$ where x is distance traveled by the particle at time t from a fixed origin.

Then $\frac{d^2x}{dt^2} = -g$ is the required differential equation.

Can you account for the minus sign on the right hand side?

Example 6

Form the differential equation from
$y = cx^2 + \sin x$ by eliminating the constant c.

Solution

The expression
$y = cx^2 + \sin x$
contains only one constant c. Then we can get differential equation of first order by differentiating the expression once and eliminating the constant c.

Let $y = cx^2 + \sin x$ (1)

$\frac{dy}{dx} = 2cx + \cos x$ (2)

To eliminate one constant we need two equations (In general to eliminate n constants we need n + 1 equations).

From (1) we can find c and the value of c is substituted in (2)
From (1), $cx^2 = y - \sin x$

$$c = \frac{y - \sin x}{x^2}$$

putting this value of c in (2)

$$\frac{dy}{dx} = 2\left[\frac{y - \sin x}{x^2}\right]x + \cos x$$

$\frac{dy}{dx} = \frac{2}{x}(y - \sin x) + \cos x$ is the required differential equation whose general solution is,
$y = cx^2 + \sin x$

Example 7
Find the differential equation whose solution is y = A cos 8x + B sin 8x.

Solution
To eliminate two constants A and B we need three equations. In addition to the given equation we need two more. These two equations are obtained by differentiating the given expression twice.

Let y = A cos 8x + B sin 8x (1)

$$y' = \frac{dy}{dx} = -8A\sin 8x + 8B\cos 8x \quad (2)$$

$$y'' = \frac{d^2y}{dx^2} = -64A\cos 8x - 64B\sin 8x \quad (3)$$

$$\frac{d^2y}{dx^2} = -64(A\cos 8x + B\sin 8x)$$

$$\frac{d^2y}{dx^2} = -64y \text{ using (1) in (3)}$$

This is the required differential equation.

1.11 Solving simple differential equations
We are going to see several methods to solve a differential equation or finding solutions of differential equations. Before that we shall see here the method of finding solutions of some simple equations using integration.

Example 8
Solve the equation, $\frac{dy}{dx} = \sin x$.

Solution
Now dy = sin x dx
Integrating both sides

$\int dy = \int \sin x dx$

then, y = - cos x + c

y + cos x = c is the required general solution of the equation.

Example 9

The differentia equation of a family of circles is given by,

$\frac{dy}{dx} + \frac{x}{y} = 0$

a) Find the equation to the family of circles.

b) If one of the family passes thro the point (1, 2) find its equation.

Solution

Let $\frac{dy}{dx} + \frac{x}{y} = 0$

$\frac{dy}{dx} = - \frac{x}{y} = 0$

y dy = - x dx

Integrating both sides,

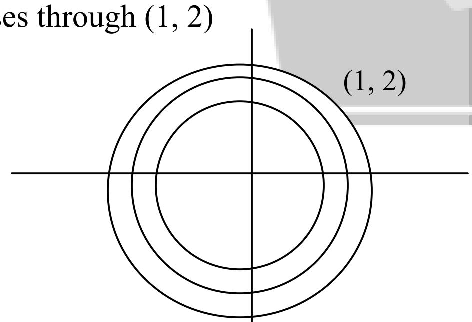
$\int y dy = -\int x dx$

$\frac{y^2}{2} = \frac{-x^2}{2} + c$

$x^2 + y^2 = A$ (where A = 2c)

This is the equation of all the circles of the family.

If it passes through (1, 2)



x = 1, y = 2 should satisfy

$x^2 + y^2 = A$

$1^2 + 2^2 = A$ or A = 5

Hence the particular solution or the equation of the particular circle is $x^2 + y^2 = 5$

Exercise 1

1. Find the order and degree of the following differential equations.

   a) $\frac{d^2y}{dx^2} + 7\left(\frac{dy}{dx}\right)^3 + 8y^2 = \sin x$

b) $\left(\frac{d^2y}{dx^2}\right)^3 + 10\left(\frac{dy}{dx}\right)^8 + 9y = \cos x$

c) $\left(\frac{d^3y}{dx^3}\right)^{\frac{1}{2}} + \left(\frac{dy}{dx}\right)^2 + y = 0$

d) $\left(\frac{dy}{dx}\right)^3 + 4\frac{dy}{dx} + 6y = e^x$

e) $\left(\frac{dy}{dx}\right)^3 - 3\frac{d^2y}{dx^2} + 7y = e^{2x}$

f) $\frac{d^2v}{dx^2}\frac{dv}{dx} + x\left(\frac{dv}{dx}\right)^2 + v = 0$

2. Find the differential equation by eliminating the constant c in the following
   a) $y = 3x^2 + 5x + c$
   b) $y = \sin x + c$
   c) $2y = 5x^2 + c$
   d) $y = Ax$
   e) $y = A \sin x$

3. Find the second order differential equation by eliminating the constants A and B .
   a) $y = Ax + B$
   b) $y = Ae^x + B$

4. Show that the differential equation associated with the primitive $y = Ax^2 + Bx + C$ is $\frac{d^3y}{dx^3} = 0$

5. Obtain the differential equation associated with the primitive $y = Ae^{2x} + Be^x + c$

6. Show that the differential equation associated with $y = Ae^x + B$ is $\frac{d^2y}{dx^2} = \frac{dy}{dx}$.

7. Find
   a) $\frac{\partial}{\partial x}(3x^2 + 5y^2 + 8xy)$
   b) $\frac{\partial}{\partial y}(3x^2 + 5y^2 + 8xy)$
   c) $\frac{\partial^2}{\partial x^2}(8x^3 + 9x^2 - 7y^2)$

d) $\frac{\partial^2}{\partial y^2}(8x^3 + 9x^2 - 7y^2)$

8. Find the general solution of the following equations

a) $\frac{dy}{dx} = 2x^2 + 3x + 5$

b) $\frac{dy}{dx} = 2e^x + \sin x$

c) $\frac{dy}{dx} = x^3 + 5x + 1$

9. Find the family of circles represented by $\frac{dy}{dx} + \frac{x}{y} = 0$

Find the particular circle of the family which passes through the point (2, -2).

Summary

You have learnt the following from this lesson:

1. Definitions of ordinary and particular differential equations.
2. The order and degree of a differential equation.
3. The meaning of the general and particular solutions of a differential equation.
4. Formation of differential equations by eliminating the arbitrary constants.
5. Finding the solution of simple differential equations.

Further Reading

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John Wiley & sons
New York Toronto. Singapore.

2. Mathematics method for Science Students
By G. Stephenson
Addison Wesley Longman Limited
Edinburgh Gate, Harlow, England, 1973.

3. Differential equations
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London Nairobi.

LESSON 2

Equations of First order – Variables Separable and Homogeneous Equations

2.1. Introduction

There are various methods for solving a first order linear differential equations. The method of "**Separation of Variables"** is the most fundamental. If the differential equation is of the form,

$$\frac{dy}{dx} = F(x, y) = \frac{f(x)}{g(y)}$$

we say that the variables are separable and obtain the solution from

g(y)dy = f(x)dx

by integrating both sides

Hence $\int g(y)dy = \int f(x)dx$

In this method, we must manage to transform the given equation in the form

g(y)dy = f(x)dx

so that the variables g(y) and dy are on one side and f(x) and dx on the other side.
Not all functions F(xy) are separable into the form f(x) dx = g (y) dy.

2.2. Objective of the lesson

By the end of this lesson you should be able to:

i. solve the first order equation using the method of separation of variables
ii. reduce a homogeneous equation to the variables separable form by using the substitution y = vx and solve the equation.
iii. Transform $\frac{dy}{dx} = \frac{ax + by + c}{px + qy + r}$ into a homogeneous equation of first order and solve this equation.

2.3. Method of separation of variables

As mentioned in the introduction, the first order linear equation

$$\frac{dx}{dy} = F(x, y)$$

is to be transformed in the form

g(y)dy = f(x)dx (2)

Only certain problems can be split into this form (2) and in such case we integrate each side to obtain the general solution. We put one constant C at the end.

Example 1

Solve the equation $\frac{dy}{dx} = \frac{2x^2}{y^3}$

Solution

We can transform the equation.

$$\frac{dy}{dx}=\frac{2x^2}{y^3} \text{ into the form}$$

$$y^3dy = 2x^2dx$$

Then $\int y^3 dy = \int 2x^2 dx$

$$\frac{y^4}{4}=\frac{2x^3}{3}+C$$

or $\quad \frac{y^4}{4}=\frac{2x^3}{3}+C$

or $\quad 3y^4 = 8x^3 + 3A$, is the required solution

Example 2

Solve $\frac{dy}{dx}=\frac{xy^2+x}{yx^2+y}$

Solution

Let $\frac{dy}{dx}=\frac{xy^2+x}{yx^2+y}$

We shall try to separate f(y) and dy on one side and f(x) and d(x) on the other side.

$$\frac{dy}{dx}=\frac{x(1+y^2)}{y(1+x^2)}$$

$$ydy=\frac{x(1+y^2)}{(1+x^2)}dx$$

$$\frac{ydy}{1+y^2}=\frac{x}{1+x^2}dx$$

$$\int\frac{ydy}{1+y^2}=\int\frac{xdx}{1+x^2} \qquad (2)$$

we can make the integrals in the form of $\int\frac{u'}{u}dx$ whose integration is *ln* u. (2) becomes

$$\frac{1}{2}\int\frac{ydy}{1+y^2}=\frac{1}{2}\int\frac{xdx}{1+x^2}$$

$$\frac{1}{2}\ln(1+y^2)=\frac{1}{2}\ln(1+x^2)+C$$

$$\ln(1+y^2)=\ln(1+x^2)+C_1, \quad (C_1 = 2C)$$

Whenever *ln* appears after integration we always call the constant as *ln* A and put A inside the logarithm to reduce the number of terms.

$$\ln(1+y^2)=\ln A(1+x^2)$$

Raising both sides to the base e,

$$e^{\ln(1+y^2)} = e^{\ln A(1+x^2)}$$

Then $(1 + y^2) = A(1 + x^2)$ is the required solution.

2.4. Solving homogeneous differential equation of first order.

First we shall define a homogeneous equation of first order and then we shall see that the homogeneous equation can be solved using the method of separation of variables when we use the substitution y = vx

2.5. Definition of homogeneous equation of first order

Consider $\frac{dy}{dx} = f(x,y)$

Suppose we put x = kx and y = ky. If f(x,y) becomes $k^n f(x,y)$ then f(x,y) is called a homogeneous function in x and y. In this section we consider homogeneous equation of order 0 so that $k^n = k^0 = 1$. In other words we consider f(kx, ky) = f(x,y) as the particular homogeneous equation.

For example

$$f(x,y) = \frac{xy}{x^2+y^2}$$

$$f(kx,ky) = \frac{kx.ky}{k^2x^2+k^2y^2}$$

$$= \frac{k^2}{k^2}\left(\frac{xy}{x^2+y^2}\right)$$

$$= \frac{xy}{x^2+y^2}$$

$$= f(x,y)$$

Here $f(kx,ky) = k^0 \frac{xy}{x^2+y^2} = f(x,y)$. Hence $\frac{xy}{x^2+y^2}$ is a homogeneous function of degree 0.

2.6. Method of solving $\frac{dy}{dx} = f(x,y)$ when f(x,y) is a homogeneous function of x and y

Let $\frac{dy}{dx} = f(x,y)$

Put the substitution y = vx

So that $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

As we shall see in the following examples the given homogeneous function will become

$$v + x\frac{dv}{dx} = f(v)$$

$$x\frac{dv}{dx} = f(v) - v$$

$$\frac{x}{dx} = \frac{f(v) - v}{dv}$$

or $$\frac{dv}{f(v) - v} = \frac{dx}{x}$$

Then $$\int \frac{dv}{f(v) - v} = \int \frac{dx}{x}$$

Thus the variables v and x are separated and the equation is solved.

Example 3

a. Show that $\frac{xy}{x^2 + y^2}$ is a homogeneous function in x and y.

b. Using the substitution y = vx transform the equation,

$$\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$$

into an equation containing v and x only.

c. Hence solve the resulting equation using the method of separation of variables. $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$

Solution

a. Let $f(x, y) = \frac{xy}{x^2 + y^2}$

$$f(kx, ky) = \frac{k^2(xy)}{k^2(x^2 + y^2)}$$
$$= f(x, y)$$

Hence $\frac{xy}{x^2 + y^2}$ is a homogeneous equation.

b. If we let y = vx, then $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

Substitution of this in the given equation, we get

$$v + x\frac{dv}{dx} = \frac{xy}{x^2 + y^2}$$ becomes

$$v + x\frac{dv}{dx} = \frac{x.xv}{x^2 + v^2x^2}$$

$$v + x\frac{dv}{dx} = \frac{v}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{v}{1+v^2} - v$$

$$x\frac{dv}{dx} = \frac{v - v\left(1 - v^2\right)}{1+v^2}$$

$$= \frac{v - v - v^3}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{-v^3}{1+v^2}$$

$$\frac{x}{dx} = \frac{-v^3}{\left(1+v^2\right)dv}$$

$$\frac{dx}{x} = -\frac{\left(1+v^2\right)dv}{v^3} \quad \text{(variables are separated)}$$

$$\int\frac{dx}{x} = -\int\left[\frac{1}{v^3} + \frac{v^2}{v^3}\right]dv$$

$$\ln A = -\int\left[v^{-3} + \frac{1}{v}\right]dv$$

$$= \frac{v^{-2}}{2} - \ln v$$

$$\ln Ax - \ln v = \frac{1}{2v^2}$$

$$\ln\frac{Ax}{v} = \frac{1}{2v^2} \quad \text{where } v = \frac{y}{x}$$

$$\ln\left(\frac{Ax^2}{y}\right) = \frac{x^2}{2y^2}$$

This is the required solution.

Example 4

Solve $\left(x^3 + y^3\right)dx - 3xy^2dy = 0$

Solution

Let $\left(x^3 + y^3\right)dx - 3xy^2dy = 0$

$$\left(x^3 + y^3\right)dx = 3xy^2dy$$

$$\frac{dy}{dx} = \frac{x^3 + y^3}{3xy^2},$$

$\frac{x^3+y^3}{3xy^2}$ is a homogeneous function of degree 0.

Let y = vx be the substitution.

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

then

$$v+x\frac{dv}{dx}=\frac{x^2+v^3x^3}{3x.v^2x^2}$$

$$v+x\frac{dv}{dx}=\frac{1+v^3}{3v^2}$$

$$x\frac{dv}{dx}=\frac{1+v^3}{3v^2}-v$$

$$=\frac{1+v^3-v(3v^2)}{3v^2}$$

$$\frac{x}{dx}=\frac{1-2v^3}{3v^2dv}$$

therefore $\frac{dx}{x}=\frac{3v^2dv}{1-2v^3}$

$$\int\frac{dx}{x}=\int\frac{3v^2dv}{1-2v^3}$$

$$\ln(Ax)=\frac{1}{-2}\int\frac{-6v^2dv}{1-2v^3}$$

$\ln(Ax)=-\frac{1}{2}\ln(1-2v^3)$ (using the substitution u = 1 – 2v^3)

$$\ln(Ax)=\ln(1-2v^3)^{-\frac{1}{2}}$$

$Ax=(1-2v^3)^{-\frac{1}{2}}$ where $v=\frac{y}{x}$

$$Ax=\left(1-\frac{2y^3}{x^3}\right)^{-\frac{1}{2}}$$

$$A^2x^2=\left(1-\frac{2y^3}{x^3}\right)^{-1}$$

$$A^2x^2=\frac{x^3}{x^3-2y^3}$$

$$A^2=\frac{x}{x^3-2y^3}$$

or $x=c(x^3-2y^3)$

is the required solution.

Example 5

Solve the equation $(x^2 - 3y^2)dx + 2xydy = 0$

Solution

Let $(x^2 - 3y^2)dx + 2xydy = 0$

or $$\frac{dy}{dx} = -\frac{(x^2 - 3y^2)}{2xy}$$

$$\frac{dy}{dx} = \frac{3y^2 - x^2}{2xy}$$

if $$f(x, y) = \frac{3y^2 - x^2}{2xy}$$

$$f(kx, ky) = \frac{3k^2y^2 - k^2x^2}{2kxky}$$

$$= \frac{k^2(3y^2 - x^2)}{k^2(2xy)}$$

$$= f(x, y)$$

Hence the given equation is homogeneous.

Letting y = vx, so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$, the given equation becomes

$$v + x\frac{dv}{dx} = \frac{3v^2y^2 - x^2}{2xvx}$$

$$= \frac{3v^2 - 1}{2v}$$

$$x\frac{dv}{dx} = \frac{3v^2 - 1}{2v} - v$$

$$x\frac{dv}{dx} = \frac{v^2 - 1}{2v}$$

$$\frac{dx}{x} = \frac{2v}{v^2 - 1}dv$$

Integrating, we find $\ln(v^2 - 1) = \ln|cx|$

Hence $$v^2 - 1 = cx$$

$$\frac{y^2}{x^2} - 1 = cx$$

$$y^2 - x^2 = cx^3$$

which is the required solution for the given equation.